

## 2-4 Deductive Reasoning

**Determine whether each conclusion is based on inductive or deductive reasoning.**

13. A dental assistant notices a patient has never been on time for an appointment. She concludes the patient will be late for her next appointment.

**SOLUTION:**

The dental assistant is basing her conclusion on a pattern of observations, so she is using inductive reasoning.

**ANSWER:**

inductive reasoning

15. If Eduardo decides to go to a concert tonight, he will miss football practice. Tonight, Eduardo went to a concert. Eduardo missed football practice.

**SOLUTION:**

The conclusion is based on a fact. So, it is deductive reasoning.

**ANSWER:**

deductive reasoning

17. Whenever Juanita has attended a tutoring session she notices that her grades have improved. Juanita attends a tutoring session and she concludes her grades will improve.

**SOLUTION:**

Juanita is basing her conclusion on a pattern of observations, so she is using inductive reasoning.

**ANSWER:**

inductive reasoning

**JUSTIFY ARGUMENTS** Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

19. **Given:** If a figure is a square, it has four right angles. Figure  $ABCD$  has four right angles.

**Conclusion:** Figure  $ABCD$  is a square.

**SOLUTION:**

If  $p$  is true, then  $q$  is true, but  $q$  being true does not necessarily mean that  $p$  is true.

A figure with 4 right angles could be a rectangle. So, the statement is invalid.

**ANSWER:**

Invalid; the figure could be a rectangle.

21. **Given:** If you leave your lights on while your car is off, your battery will die. Your battery is dead.

**Conclusion:** You left your lights on while the car was off.

**SOLUTION:**

If  $p$  is true, then  $q$  is true, but  $q$  being true does not necessarily mean that  $p$  is true.

The reason for the battery for being dead could be something else also, say it is too old. So, it is an invalid statement.

**ANSWER:**

Invalid; your battery could be dead because it was old.

23. **Given:** If 75% of the prom tickets are sold, the prom will be conducted at the country club. 75% of the prom tickets were sold.

**Conclusion:** The prom will be held at the country club.

**SOLUTION:**

If  $p \rightarrow q$  is a true statement and  $p$  is true, then  $q$ .

Here, the statement "if 75% of the prom tickets are sold, the prom will be conducted at the country club" is a true statement and 75% of the prom tickets were sold. So, the prom will be held at the country club is a valid statement by the Law of Detachment.

**ANSWER:**

valid; Law of Detachment

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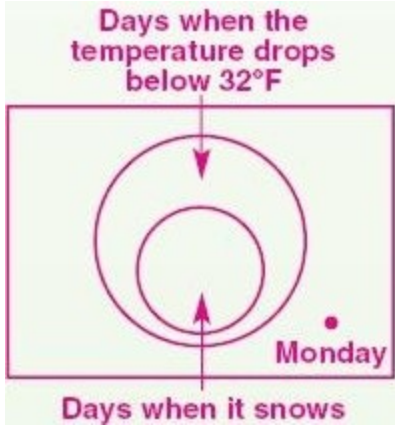
**Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning using a Venn diagram.**

25. **Given:** If the temperature drops below 32°F, it may snow. The temperature did not drop below 32°F on Monday.

**Conclusion:** It did not snow on Monday.

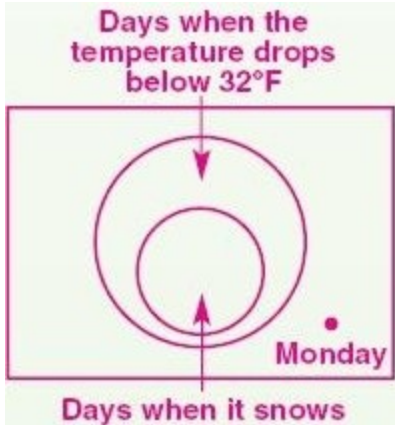
**SOLUTION:**

Monday is outside of the days when the temperature drops below 32°F, so it cannot be inside the days when it snows circle either, so the conclusion is valid.



**ANSWER:**

Valid; Monday is outside the days when the temperature drops below 32°F, so it cannot be inside the days when it snows circle either, so the conclusion is valid.



27. **Given:** Some nurses wear blue uniforms. Sabrina is a nurse.

**Conclusion:** Sabrina wears a blue uniform.

**SOLUTION:**

If  $p$  is true, then  $q$  is true, but  $q$  being true does not necessarily mean that  $p$  is true.

Sabrina could be inside just the nurses' circle or inside the intersection of the circles, so the conclusion is invalid.



**ANSWER:**

Invalid; Sabrina could be inside just the nurses' circle or inside the intersection of the circles, so the conclusion is invalid.



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29. **TRANSPORTATION** There are many types of vehicles and they are classified using different sets of criteria. Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning using a Venn diagram.

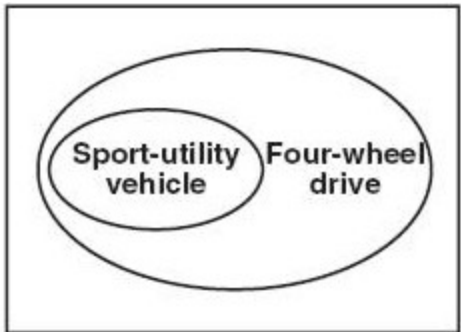
**Given:** If a vehicle is a sport-utility vehicle, then it is a four-wheel-drive car built on a truck chassis. Ms. Rodriguez has just purchased a vehicle that has four-wheel drive.

**Conclusion:** Ms. Rodriguez has just purchased a sport-utility vehicle.

**SOLUTION:**

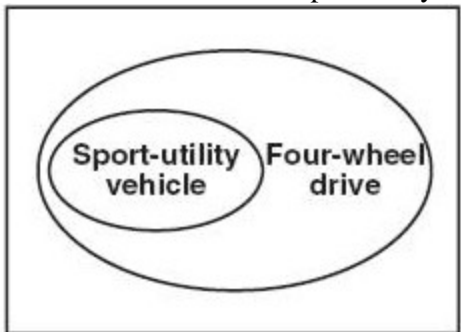
If  $p$  is true, then  $q$  is true, but  $q$  being true does not necessarily mean that  $p$  is true.

Ms. Rodriguez's car might be in the four-wheel-drive section of the diagram but not in the sport-utility vehicle section. So, the conclusion is invalid.



**ANSWER:**

Invalid; Ms. Rodriguez's car might be a four wheel drive vehicle that is not a sport-utility vehicle section.



**JUSTIFY ARGUMENTS** Use the Law of Syllogism to draw a valid conclusion from each set of statements, if possible. If no valid conclusion can be drawn, write *no valid conclusion* and explain you're reasoning.

31. If you interview for a job, then you wear a suit.  
If you interview for a job, then you will update your resume.

**SOLUTION:**

The Law of Syllogism allows you to draw conclusions from two true conditional statements when the conclusion of one statement is the hypothesis of the other. The hypothesis of one statement is not the conclusion of the other statement, so no valid conclusion can be made from the statements.

**ANSWER:**

no valid conclusion

33. If two lines are perpendicular, then they intersect to form right angles.  
Lines  $r$  and  $s$  form right angles.

**SOLUTION:**

The Law of Syllogism allows you to draw conclusions from two true conditional statements when the conclusion of one statement is the hypothesis of the other. The hypothesis of one statement is not the conclusion of the other statement, so no valid conclusion can be made from the statements.

**ANSWER:**

no valid conclusion

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35. If two lines in a plane are not parallel, then they intersect.  
If two lines intersect, then they intersect in a point.

### **SOLUTION:**

The Law of Syllogism allows you to draw conclusions from two true conditional statements when the conclusion of one statement is the hypothesis of the other. The hypothesis of one statement *is* the conclusion of the other statement, so a valid conclusion can be made from the statements. If two lines are not parallel, then they intersect in a point.

### **ANSWER:**

If two lines in a plane are not parallel, then they intersect in a point.

**Draw a valid conclusion from the given statements, if possible. Then state whether your conclusion was drawn using the Law of Detachment or the Law of Syllogism. If no valid conclusion can be drawn, write *no valid conclusion* and explain your reasoning.**

37. **Given:** If a figure is a square, then all the sides are congruent.  
Figure  $ABCD$  is a square.

### **SOLUTION:**

By the Law of Detachment if  $p \rightarrow q$  is a true statement and  $p$  is true, then  $q$ . Here, the statement "if a figure is a square, then all the sides are congruent" is a true statement and figure  $ABCD$  is a square. So, the figure  $ABCD$  has all sides congruent.

### **ANSWER:**

Figure  $ABCD$  has all sides congruent; Law of Detachment.

39. **Given:** Ballet dancers like classical music.  
If you like classical music, then you enjoy the opera.

### **SOLUTION:**

By the Law of Syllogism, If  $p \rightarrow q$  and  $q \rightarrow r$  are true statements, then  $p \rightarrow r$  is a true statement. So, if you are a ballet dancer, then you enjoy the opera.

### **ANSWER:**

If you are a ballet dancer, then you enjoy the opera; Law of Syllogism.

41. **Given:** If a polygon is regular, then all of its sides are congruent.  
All sides of polygon  $WXYZ$  are congruent.

### **SOLUTION:**

The Law of Detachment states that if  $p \rightarrow q$  is a true statement and  $p$  is true, then  $q$  is true. From the first statement,  $p$  is a polygon is regular and  $q$  is all of its sides are congruent. The second statement gives  $q$  is true. To apply the Law of Detachment, we need to have  $p$  is true to show  $q$  is true. Thus, knowing a conclusion is true does not imply the hypothesis will be true. So, no valid conclusion can be made.

### **ANSWER:**

No valid conclusion; knowing a conclusion is true does not imply the hypothesis will be true.

44. **WRITING IN MATH** Explain why the Law of Syllogism cannot be used to draw a conclusion from these conditionals.

*If you wear winter gloves, then you will have warm hands.*

*If you do not have warm hands, then your gloves are too thin.*

### **SOLUTION:**

The Law of Syllogism states that if  $p \rightarrow q$  and  $p \rightarrow r$  are true statements, then  $p \rightarrow r$  is a true statement.

From the first statement,  $p$  is "you wear winter gloves" and  $q$  is "you will have warm hands."

From the second statement,  $q$  is "you do not have warm hands". This is  $\sim q$  of the first statement. From the second statement,  $r$  is "your gloves are too thin."

The Law of Syllogism cannot be used, because the hypothesis of the second conditional is the negation of the conclusion of the first conditional. In order to use the Law of Syllogism, the conclusion of one conditional must be the hypothesis of the second conditional.

### **ANSWER:**

Sample answer: The Law of Syllogism cannot be used, because the hypothesis of the second conditional is the negation of the conclusion of the first conditional. In order to use the Law of Syllogism, the conclusion of one conditional must be the hypothesis of the second conditional.

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45. **ANALYZE RELATIONSHIPS** Use the symbols from Lesson 2-2 for conjunction and disjunction, and the symbol for implies from Lesson 2-3 to represent the Law of Detachment and the Law of Syllogism symbolically. Let  $p$  represent the hypothesis, and let  $q$  represent the conclusion.

**SOLUTION:**

By the Law of Detachment if  $p \rightarrow q$  is a true statement and  $p$  is true, then  $q$ . The symbol  $\rightarrow$  stands for "implies" and the symbol  $\wedge$  stands for "and". That is, the law can be written as

$$[(p \rightarrow q) \wedge p] \rightarrow q.$$

By the Law of Syllogism, if  $p \rightarrow q$  and  $q \rightarrow r$  are true statements, then  $p \rightarrow r$  is a true statement. This can be written as,

$$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r).$$

**ANSWER:**

Law of Detachment:  $[(p \rightarrow q) \wedge p] \rightarrow q$ .

Law of Syllogism:  $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ .

46. **ORGANIZE IDEAS** Write a pair of statements in which the Law of Syllogism can be used to reach a valid conclusion. Specify the conclusion that can be reached.

**SOLUTION:**

The Law of Syllogism states that if  $p \rightarrow q$  and  $q \rightarrow r$  are true statements, then  $p \rightarrow r$  is a true statement.

For statement (1) let  $p$  is "a student earns 40 credits" and  $q$  is "he/she will graduate from high school."

For statement (2) let  $q$  is "a student graduates from high school" and  $r$  is "he or she will receive a diploma."

(1) If a student earns 40 credits, then he/she will graduate from high school.

(2) If a student graduates from high school, then he or she will receive a diploma.

Conclusion: If a student earns 40 credits, he or she will receive a diploma.

**ANSWER:**

Sample answer: (1) If a student earns 40 credits, then he/she will graduate from high school. (2) If a student graduates from high school, then he or she will receive a diploma. Conclusion: If a student earns 40 credits, he or she will receive a diploma.

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47. **JUSTIFY ARGUMENTS** Students in Mr. Kendrick’s class are divided into two groups for an activity. Students in group A must always tell the truth. Students in group B must always lie. Jonah and Janeka are in Mr. Kendrick’s class. When asked if he and Janeka are in group A or B, Jonah says, “We are both in Group B.” To which group does each student belong? Explain your reasoning.

**SOLUTION:**

Jonah statement can be restated as, “Jonah is in group B and Janeka is in group B.”

In order for this compound statement to be true, both parts of the statement must be true. If Jonah was in group A, he would not be able to say that he is in group B, since students in group A must always tell the truth. Therefore the statement that Jonah is in group B is true.

Since students in Group B must always lie, the compound statement must be false. For the compound statement to be false, the statement that Janeka is in group B must be false. Therefore, Jonah is in group B and Janeka is in group A.

**ANSWER:**

Jonah statement can be restated as, “Jonah is in group B and Janeka is in group B.” In order for this compound statement to be true, both parts of the statement must be true. If Jonah was in group A, he would not be able to say that he is in group B, since students in group A must always tell the truth. Therefore the statement that Jonah is in group B is true. For the compound statement to be false, the statement that Janeka is in group B must be false. Therefore, Jonah is in group B and Janeka is in group A.

48. **WRITING IN MATH** Compare and contrast inductive and deductive reasoning when making conclusions and proving conjectures.

**SOLUTION:**

Sample answer: Inductive reasoning uses several specific examples to reach a conclusion, while deductive reasoning relies on established facts, rules, definitions, and/or properties to reach a conclusion. One counterexample is enough to disprove a conjecture reached using inductive or deductive reasoning. Deductive reasoning, however, is the only valid method of proving a conjecture. Inductive reasoning cannot be used to prove a conjecture.

**ANSWER:**

Sample answer: Inductive reasoning uses several specific examples to reach a conclusion, while deductive reasoning relies on established facts, rules, definitions, and/or properties to reach a conclusion. One counterexample is enough to disprove a conjecture reached using inductive or deductive reasoning. Deductive reasoning, however, is the only valid method of proving a conjecture. Inductive reasoning cannot be used to prove a conjecture.

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50. **ACT/SAT** Consider the given statements below.

- I.** If a sphere has a radius of 3 inches, then its surface area is  $36\pi$  square inches.
- II.** If a sphere has a surface area of  $36\pi$  square inches, then its volume is  $36\pi$  cubic inches.

Javier concluded that if a sphere has a radius of 3 inches, then its volume is  $36\pi$  cubic inches.

Which of the following best describes Javier's conclusion?

- F** It is false because statement I is false.
- G** It is false because statement II is false.
- H** It is false because Javier used invalid reasoning.
- J** It is valid by the Law of Detachment.
- K** It is valid by the Law of Syllogism.

### **SOLUTION:**

Determine if each of the given statements is true.

**Statement I:** The formula for the surface area of a sphere is  $S = 4\pi r^2$ . If the sphere has a radius of 3 inches, then its surface area is  $S = 4\pi(3)^2 = 36\pi$  square inches. So Statement I is true.

**Statement II:** The formula for the volume of a sphere is  $V = \frac{4}{3}\pi r^3$ . If the sphere has a surface area of  $36\pi$  square inches, then its radius is 3 inches. If a sphere has a radius of 3 inches, then its volume is  $V = \frac{4}{3}\pi(3)^3 = 36\pi$  cubic inches. So Statement II is true.

Since Statement I and Statement II are both true, answer choices F and G are eliminated.

Determine if Javier's conclusion is valid. Analyze the form of the given statements. Let  $p$  be "a sphere has a radius of 3 inches." Let  $q$  be "its surface area is  $36\pi$  square inches." Let  $r$  be "its volume is  $36\pi$  cubic inches." Javier's statement is of the form  $p \rightarrow r$ , which is the form of a valid conclusion under the Law of Syllogism. Therefore, Choice K is correct.

### **ANSWER:**

K